LETTER TO THE EDITOR

Possible Wigner solid phase transitions in a series of high-carrier-density two-dimensional hole gases

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Abstract. Transport measurements on a series of high-mobility, *high*-carrier-density ($1.0 \times 10^{11} \leq n_s \leq 1.6 \times 10^{11}$ cm^{-2}) 2D hole systems in fields up to 30 T and temperatures down to 30 mK show transitions into a very-high-resistance state at a range of Landau level filling factors (ν) near 2/7. The insulating state shows clear voltage threshold behaviour and strong temperature dependence consistent with a strongly pinned Wigner solid. We also present measurements on a gated 2D hole gas which enabled the systematic variation of the carrier density near the Wigner solid phase boundary.

As the carrier density (n_s) of a low-disorder two-dimensional (2D) conductor is lowered one expects, at sufficiently low temperatures, to see a transition to an insulating Wigner solid (WS). Such a transition was first observed for 2D electrons on the surface of helium [1]. During the last ten years interest has been concentrated upon 2D electron systems (2DES) formed in GaAs–(Ga,Al)As heterostructures. For 2D systems in zero magnetic field the onset of the WS is expected to occur at a critical value of the carrier separation a, in units of the Bohr radius, a_B. The calculated critical value of the ratio $r_s = a/a_B$ is $\simeq 37$ [2, 3]. The parameter r_s is proportional to the carrier effective mass m^* and inversely proportional to $\sqrt{n_s}$. To obtain such values of r_s would require unobtainably low densities in the low-mass n-type heterostructures. However it has recently been claimed that such a zero-field transition has been observed in the higher-mass n-type MOSFETs for which $r_s \simeq 10$, much smaller than the zero-field critical value [4]. In high magnetic fields in the extreme quantum limit when carriers are confined to the lowest Landau level the criterion for the formation of a WS is different. In the limit of *low* r_s one expects the transition to occur for a critical Landau level filling factor of $\nu_c \simeq 1/6.5$. Electrical transport, optical and RF absorption studies of GaAs–(Ga,Al)As 2DES [5], for which $r_s \simeq 2$, seem to indicate that such a transition may occur for ν_c less than and just greater than 1/5 but that at 1/5 the ground state is the fractional quantum Hall (FQH) state. Advances in the growth of high-quality 2D hole systems (2DHS) on the (311)A plane of GaAs provide a system of much larger mass and hence larger r_s, providing the prospect of observing the WS phase transition at larger ν_c than for equivalent 2DES.

Recent reports of insulating phase transitions near $\nu \simeq 1/3$ in 2DHS [8, 9, 10] identical in form to those attributed to the formation of the WS in two-dimensional electron systems, add weight to the argument that there exists a well-defined phase boundary between the normal carrier gas, the FQH liquid and the WS in an r_s–ν phase space [11, 12, 13]. In this

work we present observations of insulating transitions in a series of high-mobility 2DHS samples covering a range of carrier densities from 1.0×10^{11} cm^{-2} to 1.6×10^{11} cm^{-2}, the highest-density samples so far to exhibit such behaviour. Table 1 gives details of the devices studied.

**Table 1.** Experimental values for some parameters of the devices studied.

Sample	Dopant	Carrier density (cm^{-2})	Mobility ($T = 50$ mK) (cm^2 V^{-1} s^{-1} $\pm5\%$)	r_s
A	Si	1.0×10^{11}	210 000	10.2
B	Si	1.4×10^{11}	200 000	9.8
C	Be	1.6×10^{11}	150 000	8.7
D	Si	$1.0 - 1.4 \times 10^{11}$	500 000	$9 - 10$

Transport measurements at low currents ($\simeq 1$ nA) and low frequencies (< 10 Hz) were performed in a hybrid magnet and dilution refrigerator. Temperatures down to 40 mK were achieved and for a period of a few minutes $\simeq 30$ mK could be obtained. We estimate that the hole temperatures may be in the region of 20 mK higher than that of the mixing chamber. The measured resistances in the insulating phase are corrected for the effective input impedance of the measuring system ($\simeq 50$ MΩ)

In figure 1 we show a typical longitudinal resistance (R_{xx}) and Hall resistance (R_{xy}) as a function of magnetic field for sample C at $\simeq 35$ mK. The high quality of the sample is clearly indicated by the range of FQH minima observed and the well-quantized Hall resistance. Figure 2 shows the high-field transport (13 T $\leq B \leq 30$ T) for the three samples A, B and C. We see a dramatic insulating transition near to the 2/7 filling fraction with the resistance approaching two orders of magnitude larger than at low field for all three samples. It can be seen in figure 1 that the insulating state collapses rapidly as the temperature increases and has almost completely disappeared by 260 mK. In the lowest-density sample (A) we are able to see the emergence of the 1/5 fractional state at the higher temperatures [14] . The inset to figure 1 shows an enlarged section of the temperature dependence around the 2/7 fraction. It can clearly be seen that the resistance peak between 2/7 and 1/3 also displays temperature dependence which is not observed for lower-field R_{xx}. We attribute this to a weak re-entrance of the Wigner solid above $\nu = 2/7$. The re-entrant behaviour is not clearly displayed in the other samples contrary to expectation with their lower carrier densities [9]. We believe this reflects a longer scattering time for sample C than the zero-field mobility would seem to indicate. This is corroborated by a better defined set of fractional minima in R_{xx} for sample C than for the other samples with nominally higher zero-field mobility.

Only a limited study of the temperature dependence of the insulating phase could be performed due to the time constraints in using a hybrid magnet system. We plot the activation data for sample C in figure 3(a). The data seem to indicate the existence of two linear regions corresponding to a high- and a low-temperature activation energy. This is similar to that observed by Goldman *et al* [15] who liken this behaviour to that of samples showing fixed and variable range hopping. The figure 3(b) shows the activation energy ΔE, obtained from the low-temperature region of the activation plot, against filling factor. It is predicted that this should have the general form $\Delta E(\nu) = k_B T_0 (1 - \nu/\nu_c)$ [16] close to the insulating transition. The diagram shows a linear dependence and clearly points towards the region near $\nu = 0.29$ as being the critical filling factor; however, it fails to distinguish an ordering of this critical value with carrier density. This may again be due to the differing mobilities of the samples studied. We find that the prefactor T_0 is about twice the classical
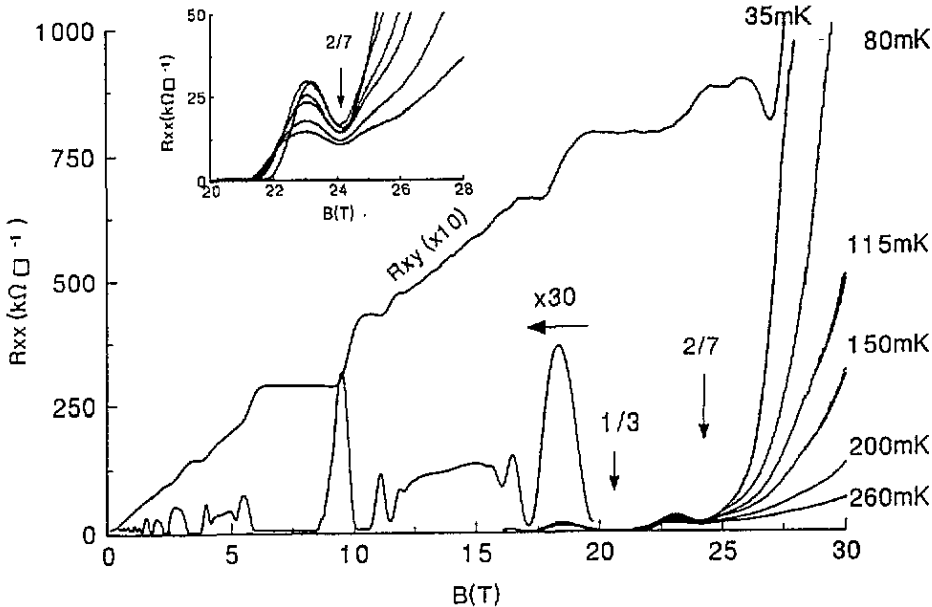
Figure 1. The magnetoresistances R_{xx} and R_{xy} for sample C up to 30 T. The R_{xx} trace below 20 T is enlarged by a factor of 30. The highest-field data show the temperature dependence of the insulating state. The inset is an enlarged section of the R_{xx} temperature dependence around the 2/7 FQH minimum.

melting temperature, which for our samples is $\simeq 650$ mK. Experiment and calculation on 2DESs yield similar values for T_0 [16]. In the limit of ν tending to zero, $\Delta E(\nu)$ should deviate from this linear dependence and tend to the classical value.

In order to examine the pinning of the insulating state the AC differential resistances of the samples were measured as a function of applied DC bias. Measurements were performed with an AC excitation current of 1 nA with the DC bias applied across the AC voltage contacts. The results are shown in figure 4. The series of full curves are for sample A taken at fixed magnetic field ($B = 17.25$ T, $\nu = 0.240$) where R_{xx} shows strong insulating behaviour. It can be seen that as one reduces the temperature a strong non-linearity develops. A plateau appears at the lowest temperatures prior to the onset of breakdown of the insulating state at an electric field of $\simeq 70$ V m^{-1}. Similar behaviour is observed for sample C at 50 mK (broken curves) and sample B at 40 mK (chain curves) which have breakdown fields of $\simeq 50$ V m^{-1} and $\simeq 75$ V m^{-1} respectively. The development of the dip at zero bias for sample C, also seen at 70 mK for sample A, is due to the input impedance of the measurement/bias system which provides a high-impedance parallel current path. The effect is quite small when the DC bias is applied across the AC voltage contacts but becomes more pronounced when applied across the AC current contacts. Razin *et al* [17] calculate the threshold field for an electron WS pinned by background donors and acceptors. They find that the charged acceptors give the dominant contribution to the strength of the pinning. We infer that in the case of a hole, WS charged donors would dominate. Substituting the background charged donor density N_D for the charged acceptor density we then estimate the threshold field from

$$E_{th} = (0.35e/\epsilon)(N_D/n_s^{1/2})(\ln(S \times 0.68N_D/n_s^{1/2}))^{-1/2}$$
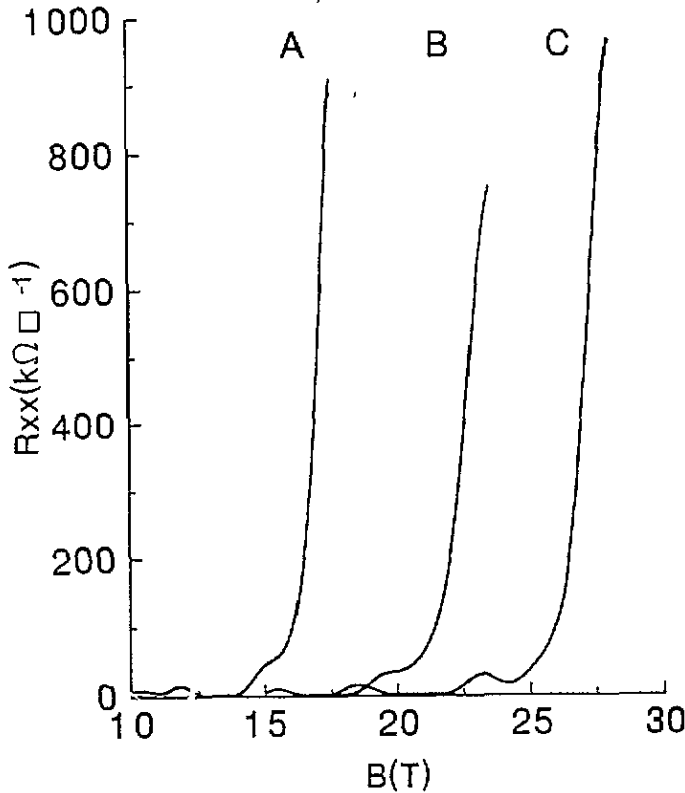
Figure 2. The insulating transition in R_{xx} at 40 mK for samples A, B and C. The strengthening of the 2/7 FQH minimum can be seen immediately prior to the insulating transition.

where S is the sample area ($\sim 10^{-6}$ m^2). Measurements on samples grown in our MBE machine give typical room temperature values of $N_D \sim 10^{19}$ to 10^{20} m^{-3} (see [18] for example: note that Si is incorporated as an acceptor on the (311)A plane). This gives values of $E_{th} \simeq 40$ to 350 V m^{-1} for our samples. The observed breakdown fields are therefore consistent with those expected for a WS.

Measurements on a gated 2DHS (sample D) enabled the systematic variation of the carrier density near the Wigner solid phase boundary. An evaporated Au front gate was applied over a 1 μm thick layer of polyimide providing a gate isolation of better than 10^{12} Ω. Figure 5 shows the longitudinal resistance R_{xx} against filling factor as the carrier density is reduced. Measurements on an identical sample at 300 mK indicate that the mobility of the sample is unaffected by the gate bias within the range of biases used. The 2/7 fraction weakens with decreasing carrier density whilst the transition into the insulating phase appears to move to larger ν. This behaviour is consistent with that of the equivalent fixed carrier densities for samples A, B and C (figure 2). It is not clear whether the weakening of the 2/7 minimum is related to an increase in the critical filling factor associated with the insulating phase or a weakening of the FQH state due to a reduction in the magnitude of the magnetic field. Both processes may be occurring simultaneously as might also be the case with the deepening of the 2/7 states seen in figure 2.

That the 2DHG should show a much larger ν_c than the 2DEG is not surprising because the carrier effective mass, m^*, and thus the Landau level mixing is very much greater. Such
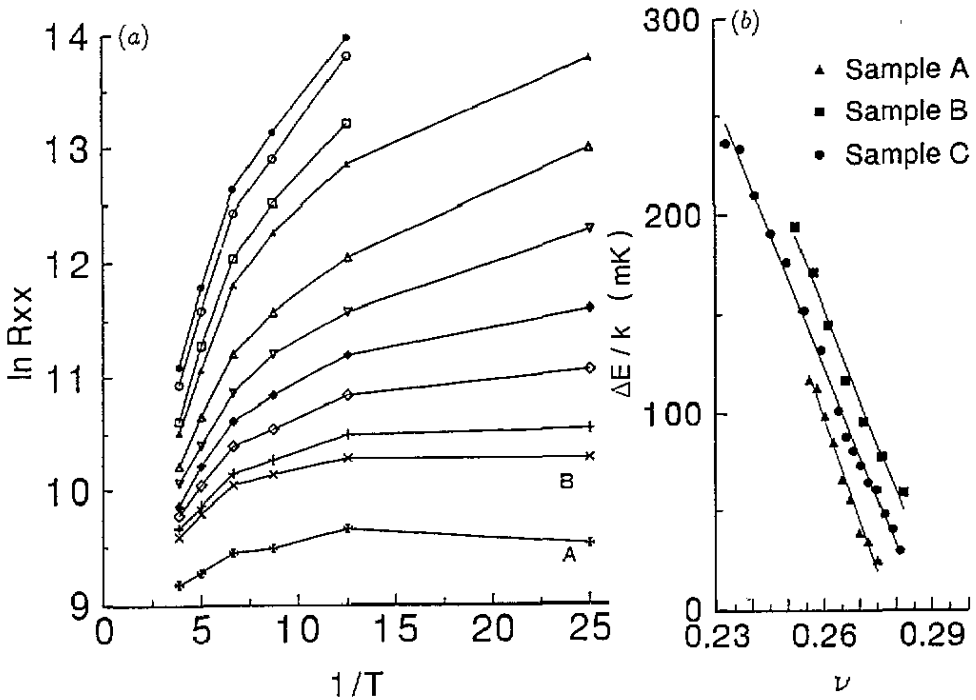
Figure 3. The temperature dependence $\ln(R_{xx})$ for sample C is shown in figure 3(a). Trace A is at the resistance maximum between filling factors 2/5 and 1/3. Trace B is at the resistance maximum between filling factors 1/3 and 2/7. The other curves are for filling factors 0.221 ($\bullet$), 0.224 (o), 0.232 ($\square$), 0.236 ($\blacktriangle$), 0.245 ($\triangle$), 0.250 ($\triangledown$), 0.255 ($\blacklozenge$), 0.260 ($\lozenge$) and 0.265 (+). Figure 3(b) shows the approximate activation energy ΔE determined from the low-temperature activation data plotted against filling factor.

mixing can lower the energy of the WS with respect to the normal or FQH state [13]. At high fields the competition between normal carrier state and WS is best thought of as dependent upon λ_1, the ratio of the Coulombic energy to the cyclotron energy. Similarly the FQH/WS transition should occur at a critical value of λ_2, the ratio of the FQH energy gap to the cyclotron energy. However, $\lambda_1 = r_s \nu$ and $\lambda_2 \simeq r_s \nu^{1/2}$, so for a given value of ν both phase boundaries will occur at a critical value of r_s. Thus it seems possible to construct a phase diagram in an r_s–ν phase space. Platzman *et al*[11] indicate the form one might expect for the normal-carrier-state FQH/WS phase diagram in an r_s–ν phase space. Recent calculations of the phase boundaries [3, 12, 13, 19] predict that ν_c should be significantly increased for $r_s \simeq 10$ and that the effects of both disorder and finite temperature will further increase ν_c.

The dispersion relationships for the confined holes on the (311)A plane have recently been calculated and measured [20, 21]. The lowest sub-band is essentially isotropic but shows strong non-parabolicity. The cyclotron mass has been determined to high accuracy in our material [22] and shows the expected strong field dependence of m^*. At $B = 4.67$ T, which is close to the insulating transition of Santos *et al* [8,9], $m^* = 0.304m_e$ in agreement with their value [8]. By 18.2 T, however, this has risen to $0.389m_e$ and extrapolation to $B \geq 22$ T gives $m^* \simeq 0.40m_e$. This gives values of r_s of ~8–10 for our samples. Previous work on the 2DEGs has only investigated a small range of values around $r_s \simeq 2$ for which $\nu_c \simeq 0.22$. Santos *et al* [9] have found $\nu_c \simeq 2/7$ to $\geq 1/3$ for samples with $r_s \simeq 9$–15 (when calculated from our measured masses). It thus seems highly plausible that a Wigner
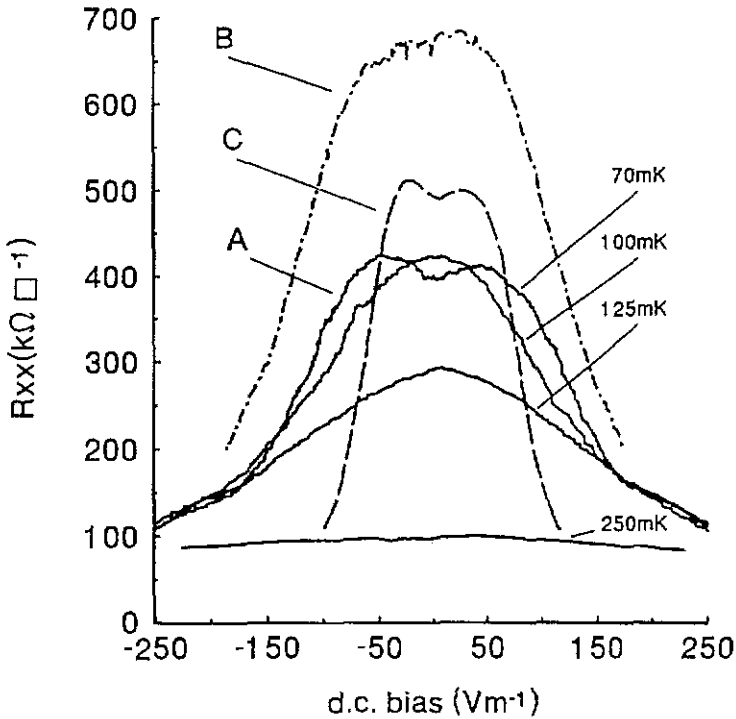
Figure 4. The AC differential resistance R_{xx} as a function of applied DC electric field for samples A (full curve, $\nu = 0.240$, $T = 70$ mK), B (broken curve, $\nu = 0.253$, $T = 40$ mK) and C (chain curve, $\nu = 0.231$, $T = 40$ mK). Also shown is the temperature dependence of the pinning for sample A for 70, 100, 125 and 250 mK.

transition should occur in our samples at a value intermediate between those observed for low-density 2DHSs and low-mass 2DESs.

To conclude we observe a strong insulating transition in the longitudinal resistance in each of our samples ranging in density from 1.0×10^{11} cm^{-2} to 1.6×10^{11} cm^{-2}. The transitions consistently group near the 2/7th filling factor irrespective of mobility or absolute magnetic field strength with one sample indicating a weak re-entrance of the insulating state around the 2/7 fraction. The insulating states show a very strong temperature dependence which is similar to that expected for Wigner crystallization and that seen in 2DES. Differential resistance measurements show clear non-linear behaviour with threshold fields of $\simeq 60$ V m^{-1}. This is again consistent with theory and similar to the behaviour taken to indicate the occurrence of a Wigner solid in 2DES. The overall non-sample-specific behaviour would be highly unlikely if the insulating transition were the result of simple single-particle magnetic localization of the carriers. It can, however, be explained consistently in terms of the theory of the formation of the Wigner solid in a high-mass system.
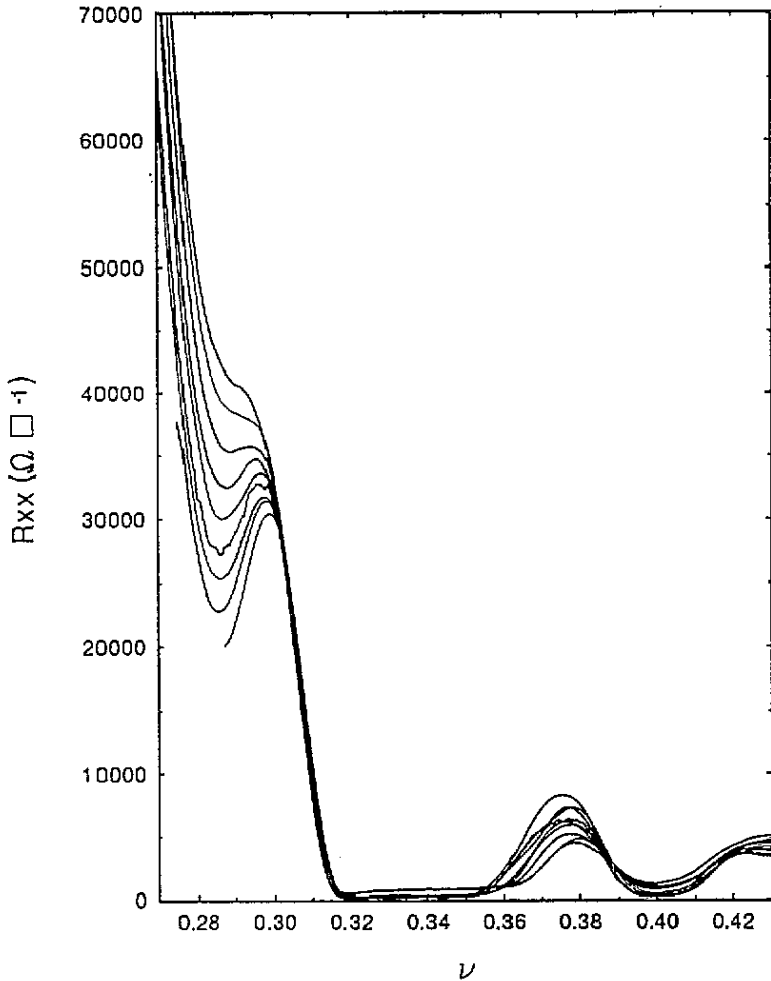
Figure 5. R_{xx} magnetoresistance for the gated sample $\mathcal{D}$ against filling factor at 40 mK. The $\nu = 2/7$ minima weakens as the carrier density is reduced linearly from 1.4×10^{11} cm^{-2} to 1.0×10^{11} cm^{-2}.

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